

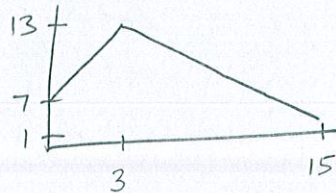
A person's velocity (in meters per second) at time  $t$  (in seconds) is given by  $v(t) = \begin{cases} 2t+7, & 0 \leq t \leq 3 \\ 16-t, & 3 \leq t \leq 15 \end{cases}$  SCORE: \_\_\_\_ / 5 PTS

[a] Find the exact distance the person travelled from time  $t = 0$  seconds to  $t = 15$  seconds.

**NOTE: You must show the arithmetic expression that you used to get your answer.**

You may only use techniques discussed in sections 5.1 and 5.2.

$$\begin{aligned} & \left(\frac{1}{2}\right) \frac{7+13}{2} \cdot 3 + \frac{13+1}{2} \cdot 12 \left(\frac{1}{2}\right) \\ & = 10 \cdot 3 + 7 \cdot 12 \\ & = 30 + 84 = 114 \text{ METERS} \left(\frac{1}{2}\right) \end{aligned}$$

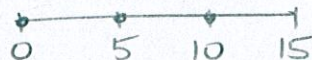


[b] Estimate the distance the person travelled from time  $t = 0$  seconds to  $t = 15$  seconds using three subintervals and left endpoints.

**NOTE: You must show the arithmetic expression that you used to get your answer.**

$$\Delta t = \frac{15-0}{3} = 5$$

$$\begin{aligned} & v(0)\Delta t + v(5)\Delta t + v(10)\Delta t \\ & = 7 \cdot 5 + 11 \cdot 5 + 6 \cdot 5 \end{aligned}$$



$$\begin{aligned} & \left(\frac{1}{2}\right) (7+11+6) \cdot 5 \left(\frac{1}{2}\right) \\ & = 24 \cdot 5 = 120 \text{ METERS} \left(\frac{1}{2}\right) \end{aligned}$$

The graph of function  $f$  is shown on the right.

The graph consists of a diagonal line, arcs of 2 circles, then another diagonal line.

SCORE: \_\_\_\_ / 4 PTS

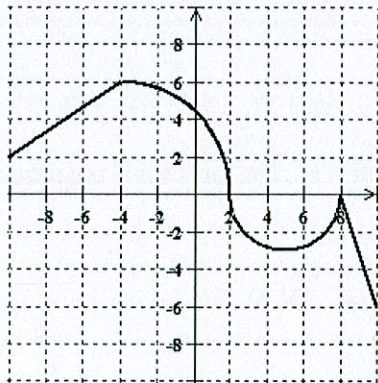
[a] Evaluate  $\int_{-10}^{10} f(x) dx$ .

NOTE: You must show the arithmetic expression that you used to get your answer.

$$\begin{aligned} & \left(\frac{1}{2}\right) \frac{2+6}{2} \cdot 6 + \left(\frac{1}{2}\right) \frac{1}{4} \pi (6)^2 - \frac{1}{2} \pi (3)^2 - \frac{1}{2} 2 \cdot 6 \\ &= 4 \cdot 6 + 9\pi - \frac{9}{2}\pi - 6 \quad \left(\frac{1}{2}\right) \quad \left(\frac{1}{2}\right) \\ &= 18 + \frac{9}{2}\pi \quad \left(\frac{1}{2}\right) \end{aligned}$$

[b] Evaluate  $\int_{-10}^8 f(x) dx$ .

$$-\int_{-10}^8 f(x) dx = -\left[4 \cdot 6 + 9\pi - \frac{9}{2}\pi\right] = -(24 + \frac{9}{2}\pi) = -24 - \frac{9}{2}\pi$$





Using the limit definition of the definite integral, and right endpoints, find  $\int_{-1}^5 (4x^2 + 8x) dx$ .

SCORE: \_\_\_\_ / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{5 - (-1)}{n} = \frac{6}{n}$$

$$\textcircled{1} \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(-1 + \frac{6i}{n}\right) \frac{6}{n} \textcircled{1}$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left[ 4\left(-1 + \frac{6i}{n}\right)^2 + 8\left(-1 + \frac{6i}{n}\right) \right] \textcircled{1\frac{1}{2}}$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left[ 4\left(1 - \frac{12i}{n} + \frac{36i^2}{n^2}\right) - 8 + \frac{48i}{n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left[ 4 - \frac{48i}{n} + \frac{144i^2}{n^2} - 8 + \frac{48i}{n} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left[ -4 + \frac{144i^2}{n^2} \right] \textcircled{2}$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} \left[ \underbrace{-4n}_{\textcircled{1}} + \frac{144}{n^2} \underbrace{\frac{n(n+1)(2n+1)}{6}}_{\textcircled{1}} \right] \textcircled{\frac{1}{2}}$$

$$= \lim_{n \rightarrow \infty} 6 \left[ -4 + 24\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) \right] \textcircled{1}$$

$$= 6[-4 + 24 \cdot 1 \cdot 2]$$

$$= 6(44)$$

$$= \underline{264} \textcircled{1} \text{ ONLY IF YOU USED THE } \lim_{n \rightarrow \infty} \sum_{i=1}^n \text{ DEFINITION + METHOD}$$

Evaluate  $\int_0^7 (|x-5| - 4\sqrt{49-x^2}) dx$  using the properties of definite integrals and interpreting in terms of area. SCORE: \_\_\_\_ / 5 PTS

**NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.**

$$\begin{aligned}
 \textcircled{2} \int_0^7 |x-5| dx - 4 \int_0^7 \sqrt{49-x^2} dx &= \underbrace{\frac{1}{2} \cdot 5 \cdot 5}_{\textcircled{\frac{1}{2}}} + \underbrace{\frac{1}{2} \cdot 2 \cdot 2}_{\textcircled{\frac{1}{2}}} - \underbrace{4 \cdot \frac{1}{4} \pi (7)^2}_{\textcircled{\frac{1}{2}}} \\
 &= \underbrace{\frac{29}{2} - 49\pi}_{\textcircled{\frac{1}{2}}}
 \end{aligned}$$

